# Accuracy and Performance of FFT Software Libraries

HIGH PERFORMANCE KERNELS LLC

## 1 INTRODUCTION

This is an update to the paper by Caprioli and Jenkins [1] to show the latest accuracy and performance data for `hpk::fft` version 0.5.0 on both `x86_64` and `aarch64`. Please see our original paper for background and discussion.

## 2 GENERAL INFORMATION

For comparison on Intel hardware, we use the Intel Math Kernel Library version 2025.0 for single and double precision and the Intel Integrated Performance Primitives version 2022.0 for half precision. These library versions are packaged for download as part of the oneAPI BaseKit version 2025.0.0. At the time of this writing, MKL does not support half precision FFTs at all, and IPP has only limited support. In particular, the functions `ippsDFT{Fwd,Inv}_Direct_CToC_16fc` therein do not support scaling, do not support in-place computations, and are limited to the 58 specific transform lengths listed in Table 1. For single and double precision, we use the 400 lengths listed in Table 2.

We also compare against FFTW3, using version `3.3.10-1` as built and distributed in Debian 12 for `amd64` and version `3.3.8-10.amzn2023.0.2` as built by Amazon Web Services (AWS) and distributed in Amazon Linux for `aarch64`. Neither supports half precision.

Results for `x86_64` were obtained on a system having an Intel Xeon `w7-2495X` processor (formerly Sapphire Rapids) with 24 cores. The operating system was Debian 12.

Results for `aarch64` were obtained on an AWS EC2 compute instance having a Graviton 3E processor. Only a single virtual CPU was provisioned. The operating system was Amazon Linux 2023.6.20241121.

## 3 ACCURACY RESULTS

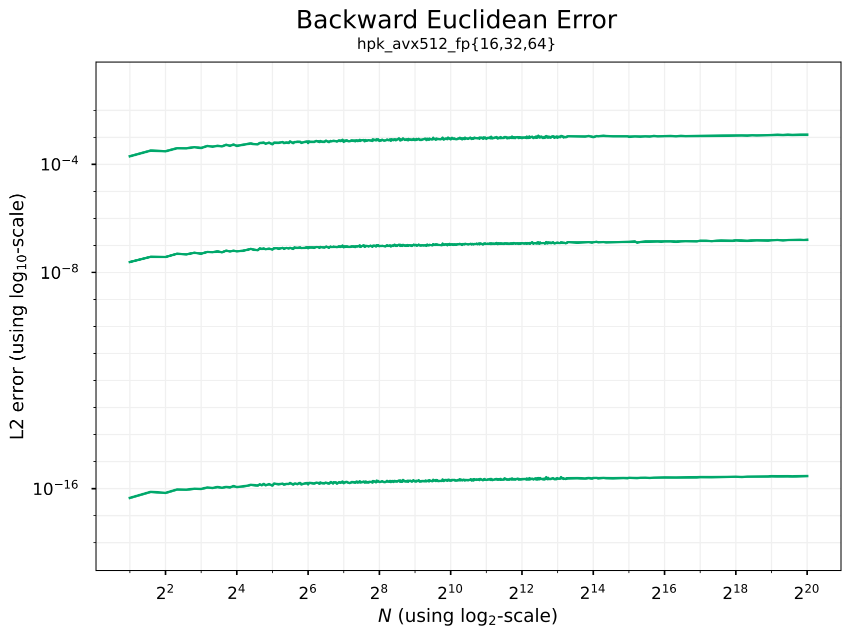
The following is a summary of `hpk::fft` accuracy given as a ratio to other libraries, where

$$\text{accuracy} = \frac{\text{error}_{\texttt{other}}}{\text{error}_{\texttt{hpk::fft}}}.$$

Thus, values greater than 1.0 indicate that `hpk::fft` is the more accurate. The values in the table below are the geometric mean of the tested transforms. Note that for the IPP comparison only the lengths in Table 1 were used. The FFTW and MKL results for single and double precision use the longer list of sizes in Table 2.

| hpk::fft | other | precision | ratio |
|---|---|---|---|
| hpk_avx2 | fftw_deb12 | fp32 | 1.105 |
| | | fp64 | 1.119 |
| | mkl_avx2 | fp32 | 1.078 |
| | | fp64 | 1.243 |
| hpk_avx512 | ipp_avx512 | fp16 | 1.041 |
| | mkl_avx512 | fp32 | 1.201 |
| | | fp64 | 1.404 |
| hpk_sve256 | fftw_aws | fp32 | 1.023 |
| | | fp64 | 1.054 |

Below, we show the error for half, single, and double precision on a single graph using a log scale for both axes. It's not possible to see any details at this high level, but it allows everything to fit and gives the overall picture. Only hpk::fft data is shown, so all 400 lengths from Table 2 are used for all three precisions.



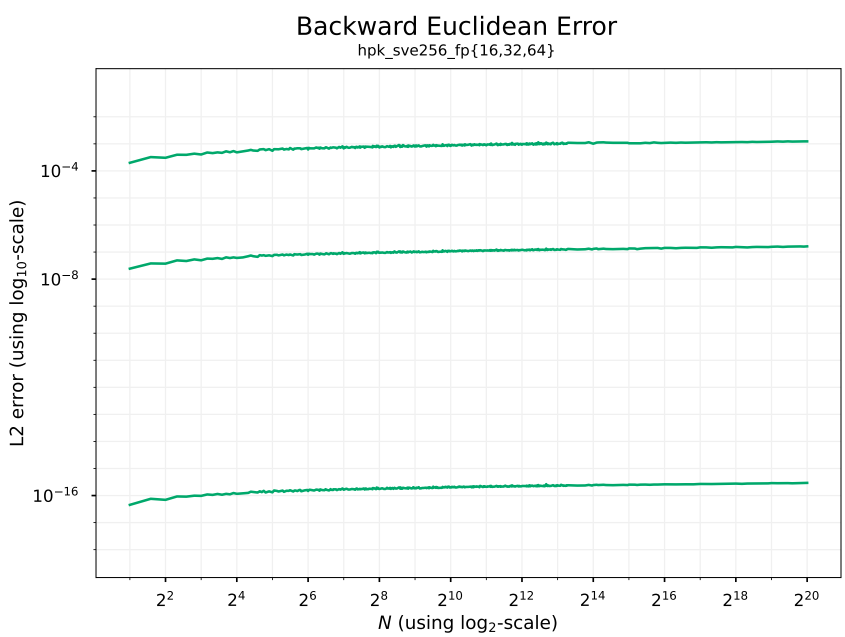




Fig. 1. Error for hpk::fft on architectures x86_64 (top) and aarch64 (bottom)

In all of the graphs that follow, we compare `hpk::fft` to some other library. The $y$-axis uses a linear scale for the error, and the $x$-axis is the transform length in strictly increasing order. The results are spaced equally apart; there is not a uniform horizontal scale. A tic mark is placed at every power-of-two length.

### 3.1 Half Precision

In Figure 2, we show the error for the 58 lengths supported by IPP, which are listed in Table 1.

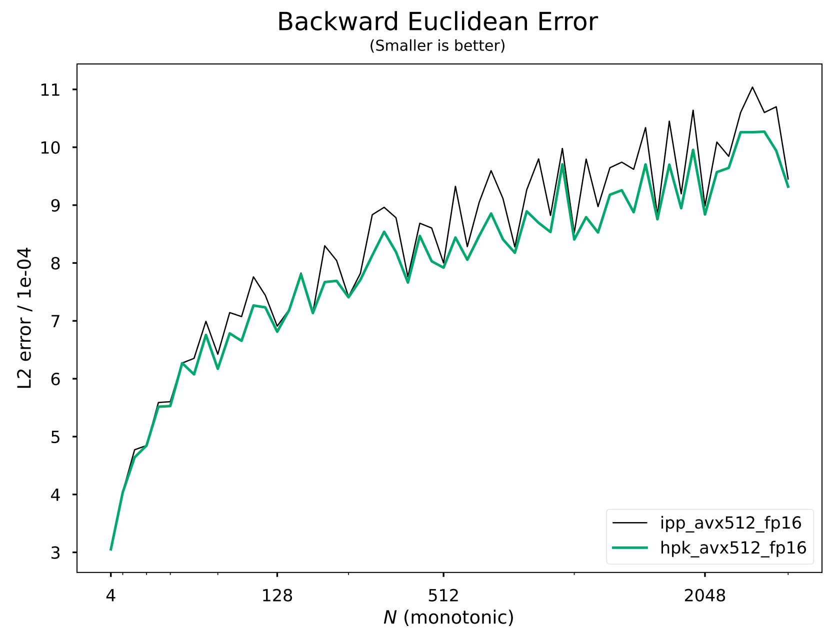



Fig. 2. Error for AVX512 half precision vs. IPP

The accuracy of `hpk::fft` is better than or equal to the accuracy of IPP at every point shown above.

## 3.2 Single Precision

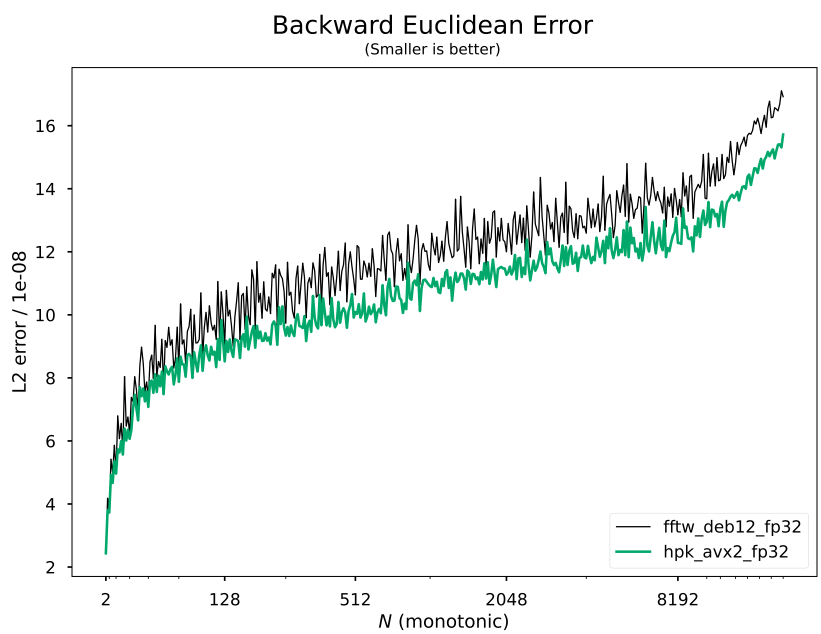



Fig. 3. Error for AVX2 single precision vs. FFTW

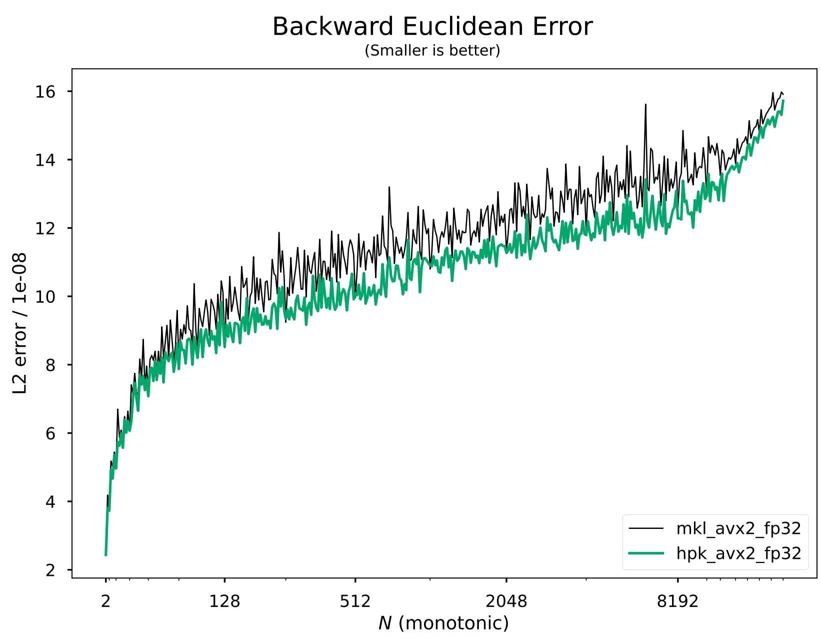



Fig. 4. Error for AVX2 single precision vs. MKL

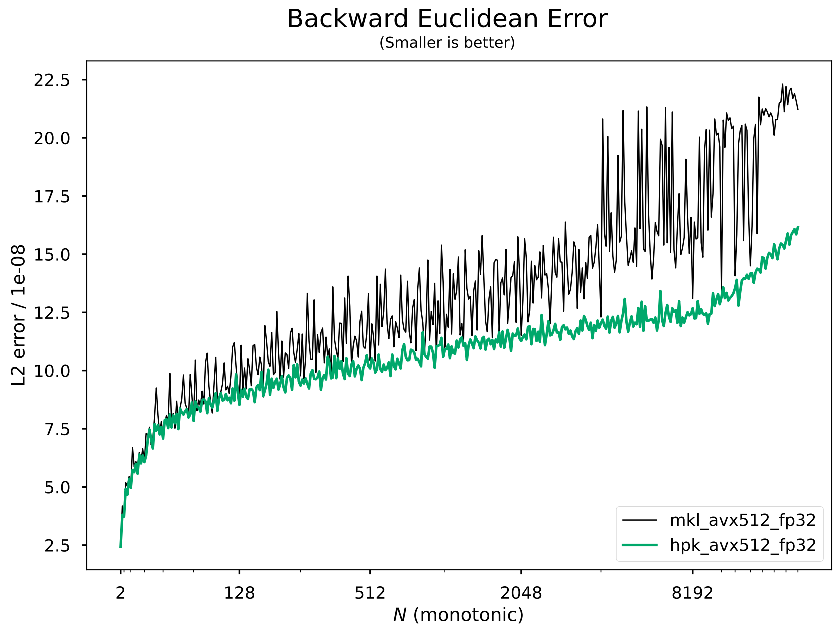



Fig. 5. Error for AVX512 single precision vs. MKL

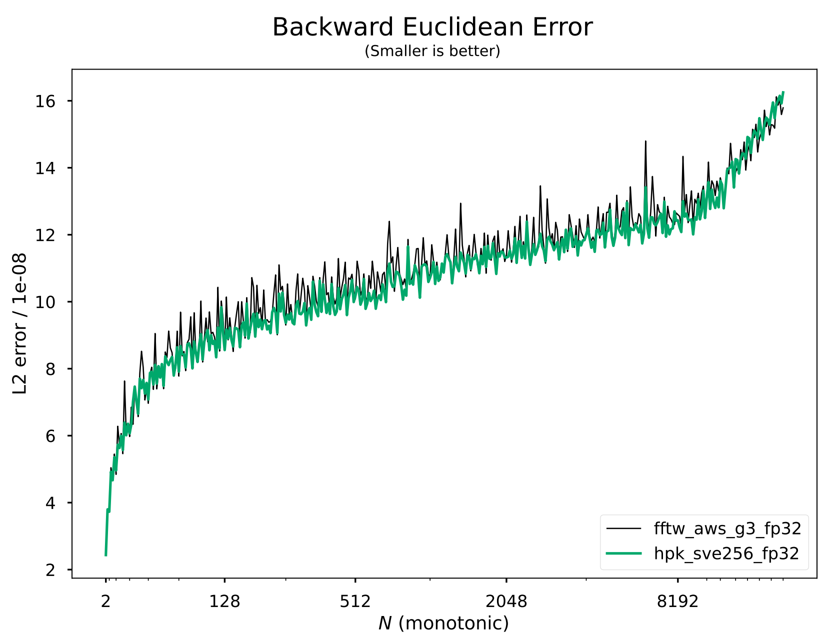



Fig. 6. Error for AArch64 SVE256 single precision vs. FFTW

## 3.3 Double Precision

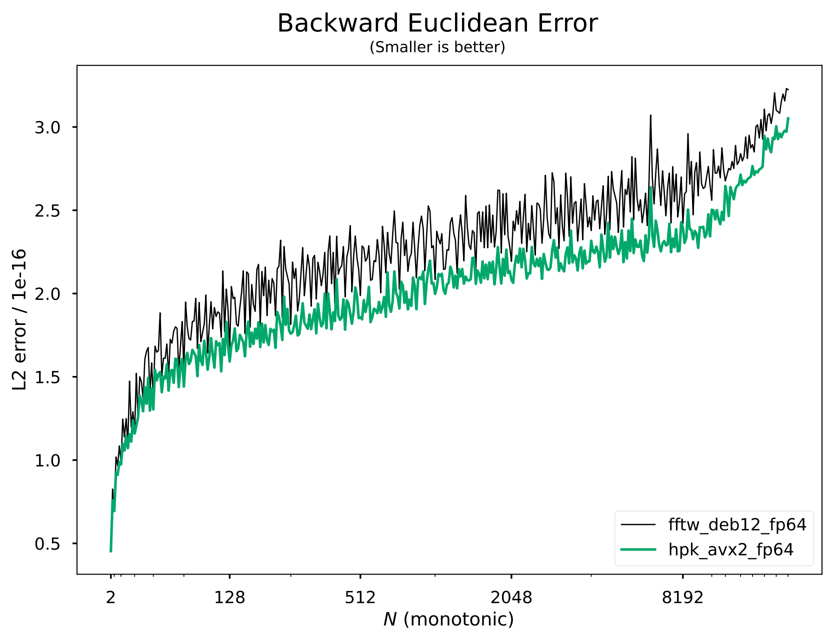



Fig. 7. Error for AVX2 double precision vs. FFTW

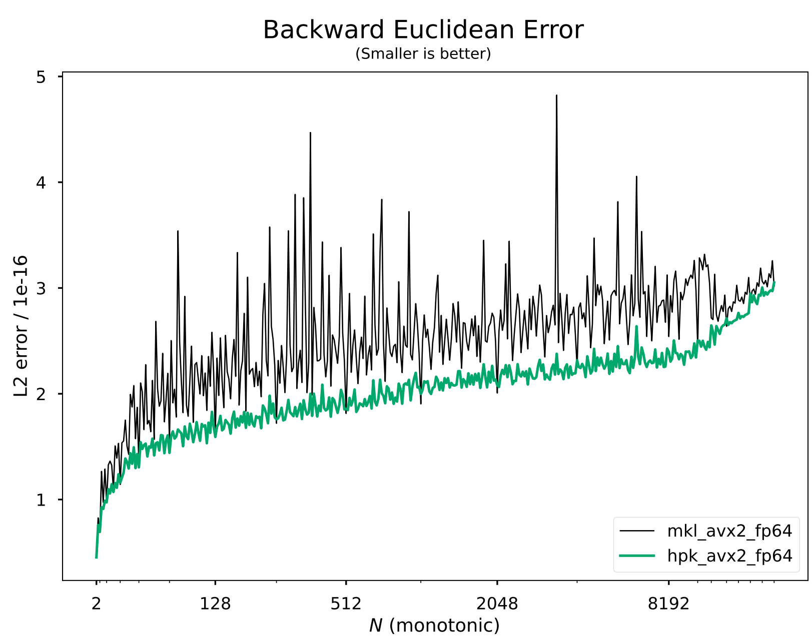



Fig. 8. Error for AVX2 double precision vs. MKL

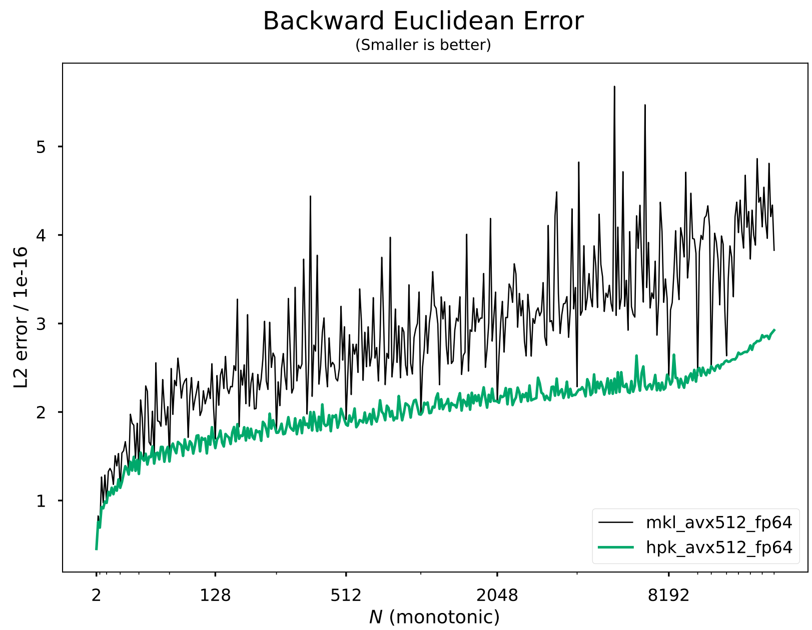



Fig. 9. Error for AVX512 double precision vs. MKL

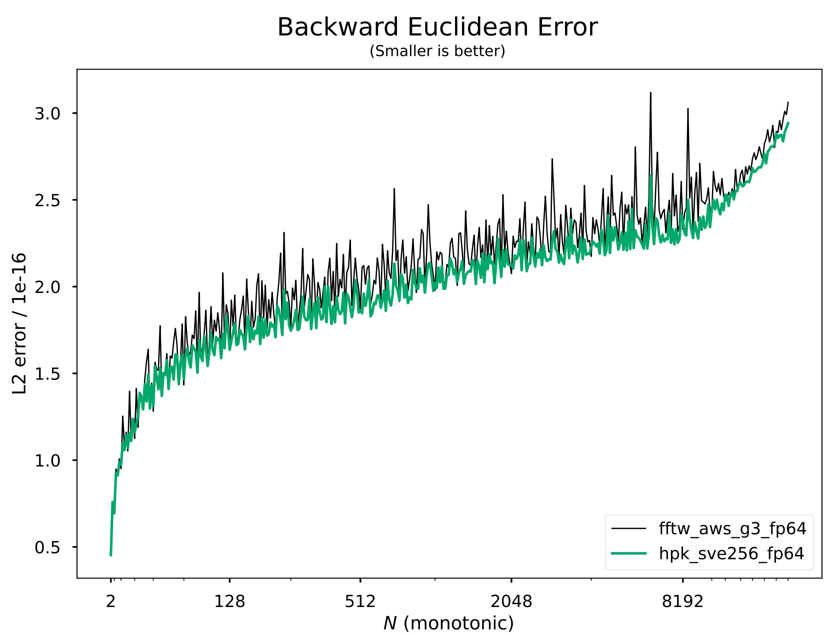



Fig. 10. Error for AArch64 SVE256 double precision vs. FFTW

## 4 PERFORMANCE RESULTS

For each transform length, four FFTs are computed, and the graphs are created after taking the geometric mean of the four performance ratios. For single and double precisions, the four test cases are the cross product of forward and backward, in-place and out-of-place. For half precision, Intel IPP only supports out-of-place computations, so we measured and compared only that, running the forward and backward transforms twice each. Unless otherwise noted, the batch size is one, i.e., a single sequence of length $N$ is transformed. In the graphs which follow, the suffix `_b7` indicates that a batch size of seven was used.

Only single-threaded performance is measured. The sequential code path is selected for `hpk::fft` by making the factory with the configuration entry `{hpk::Parameter::threads, 1}`, and the same is done for MKL by setting the environment variable `OMP_NUM_THREADS=1`.

Similarly, we measure AVX2 performance for `hpk::fft` by making the factory with the configuration entry `hpk::Architecture::avx2` and for MKL by setting `MKL_ENABLE_INSTRUCTIONS=AVX2`. Otherwise, each of these libraries automatically detects the hardware architecture as AVX512.

The following is a summary of `hpk::fft` performance given as a ratio to other libraries, where

$$\text{performance ratio} = \frac{\text{time}_{\texttt{other}}}{\text{time}_{\texttt{hpk::fft}}}.$$

Therefore, results greater than 1.0 indicate that `hpk::fft` has better performance.

| `hpk::fft` | `other` | precision | batch | ratio |
|---|---|---|---|---|
| hpk_avx2 | fftw_deb12 | fp32 | 1 | 2.240 |
| | | fp64 | 1 | 1.529 |
| | mkl_avx2 | fp32 | 1 | 1.785 |
| | | fp64 | 1 | 1.622 |
| hpk_avx512 | ipp_avx512 | fp16 | 1 | 1.224 |
| | | | 7 | 1.295 |
| | mkl_avx512 | fp32 | 1 | 1.431 |
| | | fp64 | 1 | 1.350 |
| hpk_sve256 | fftw_aws | fp32 | 1 | 3.891 |
| | | fp64 | 1 | 2.181 |

## 4.1 Half Precision

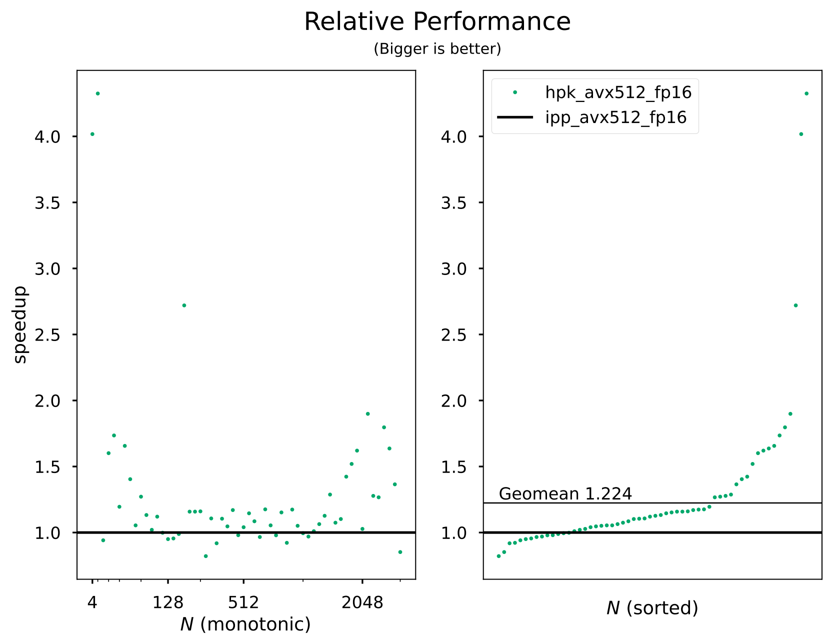



Fig. 11. Performance for AVX512 half precision vs. IPP

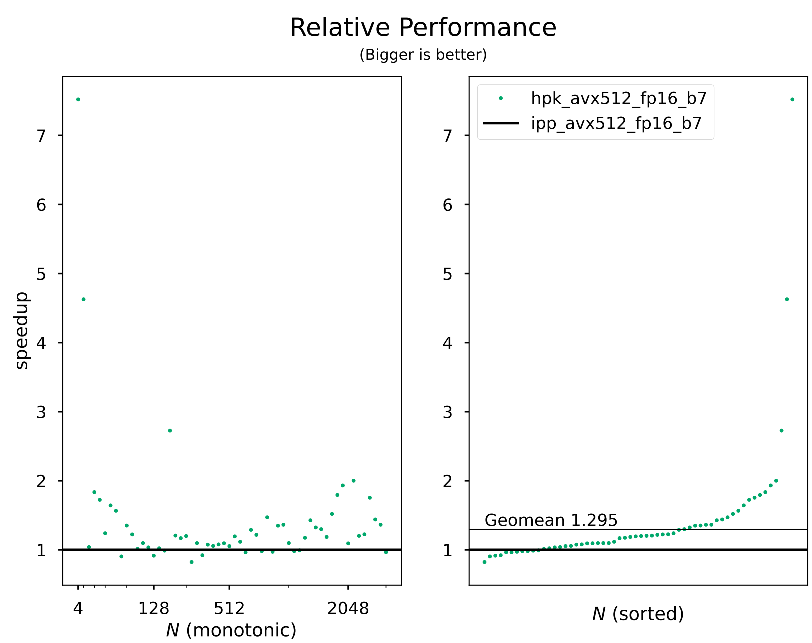



Fig. 12. Performance for AVX512 half precision vs. IPP for a batch of seven transforms

## 4.2 Single Precision

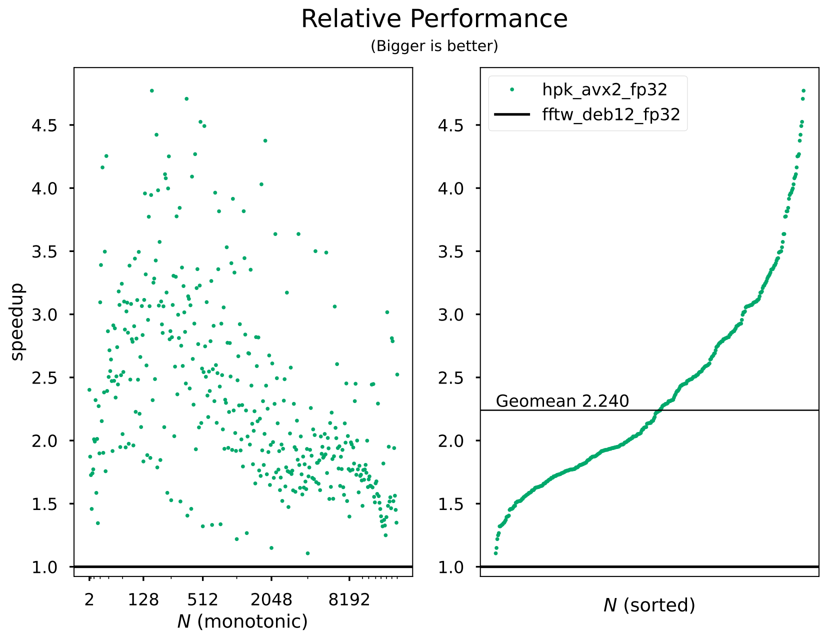



Fig. 13. Performance for AVX2 single precision vs. FFTW

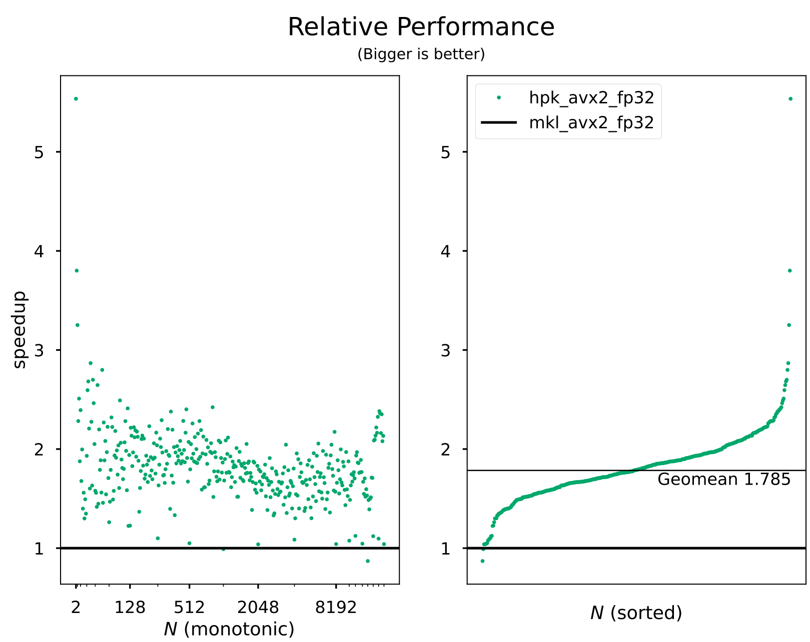



Fig. 14. Performance for AVX2 single precision vs. MKL

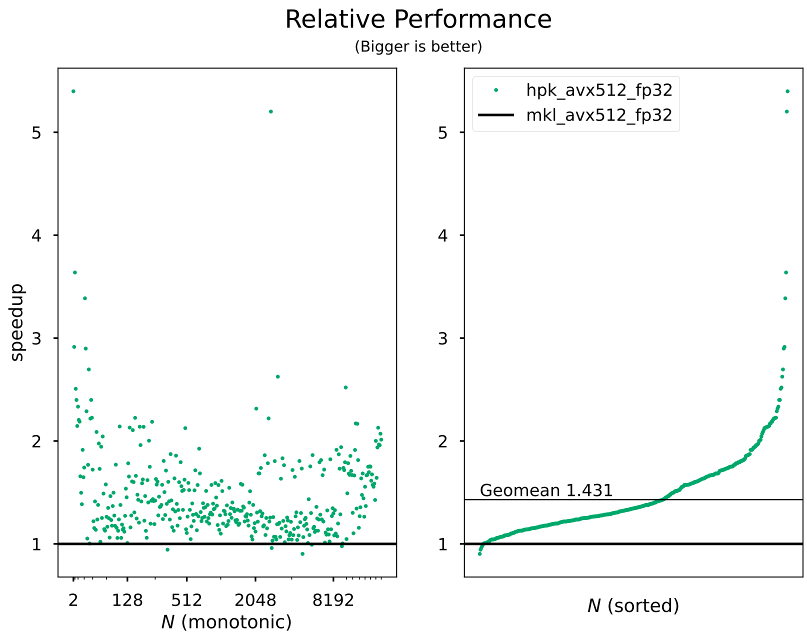



Fig. 15. Performance for AVX512 single precision vs. MKL

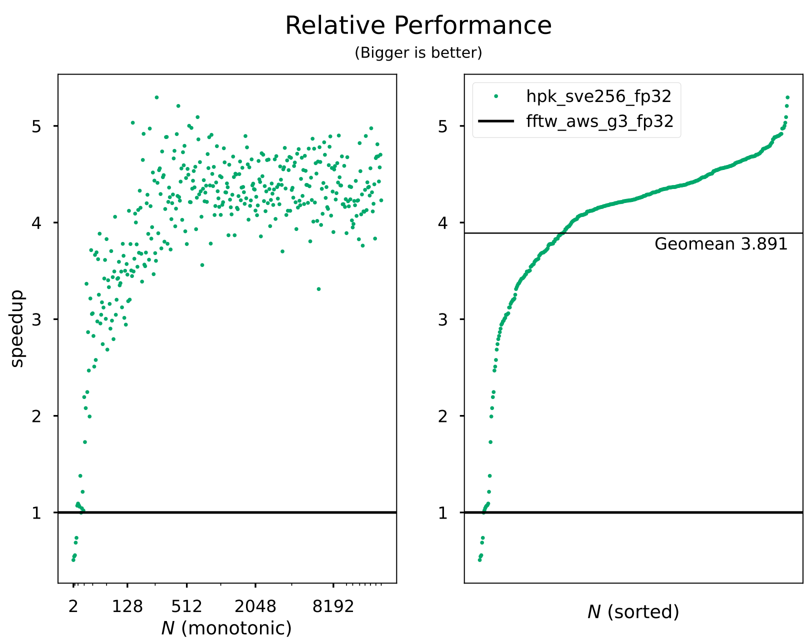



Fig. 16. Performance for AArch64 SVE256 single precision vs. FFTW

## 4.3 Double Precision

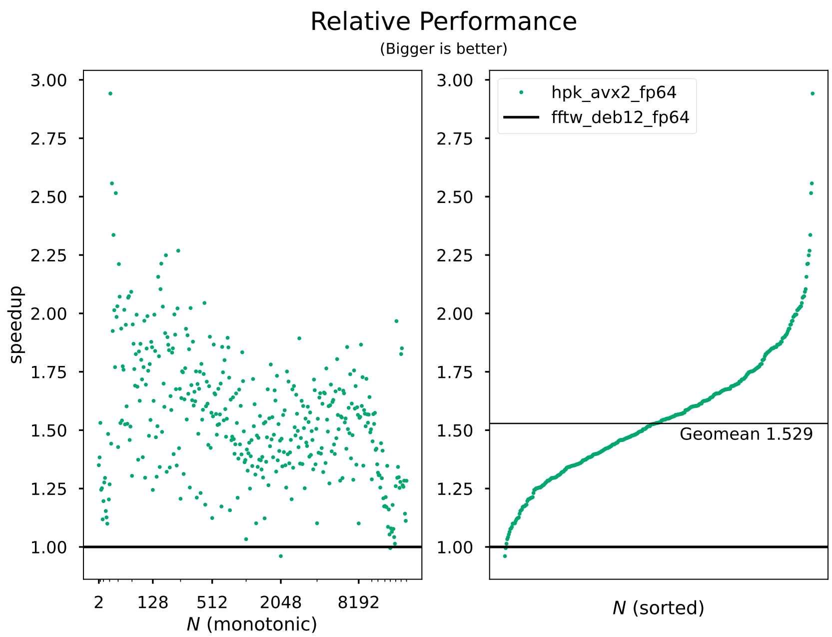



Fig. 17. Performance for AVX2 double precision vs. FFTW

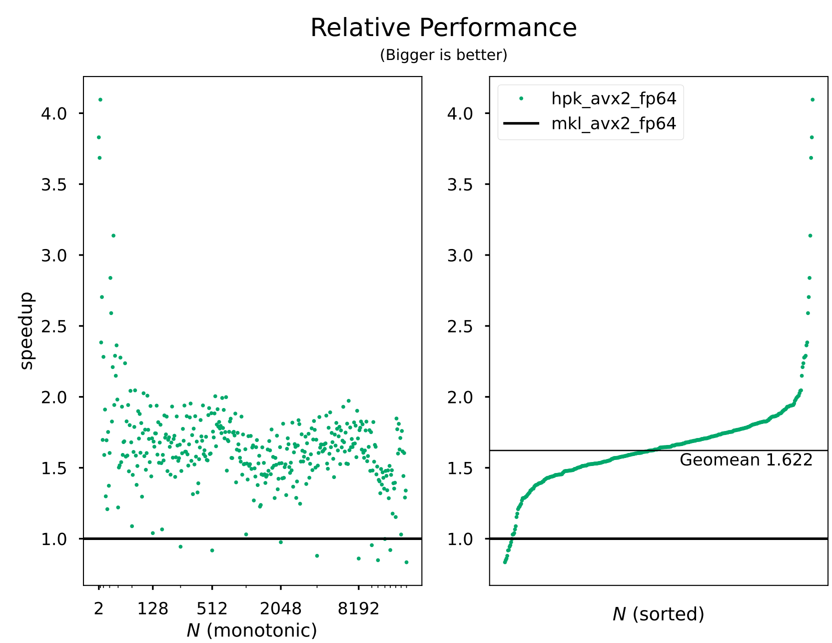



Fig. 18. Performance for AVX2 double precision vs. MKL

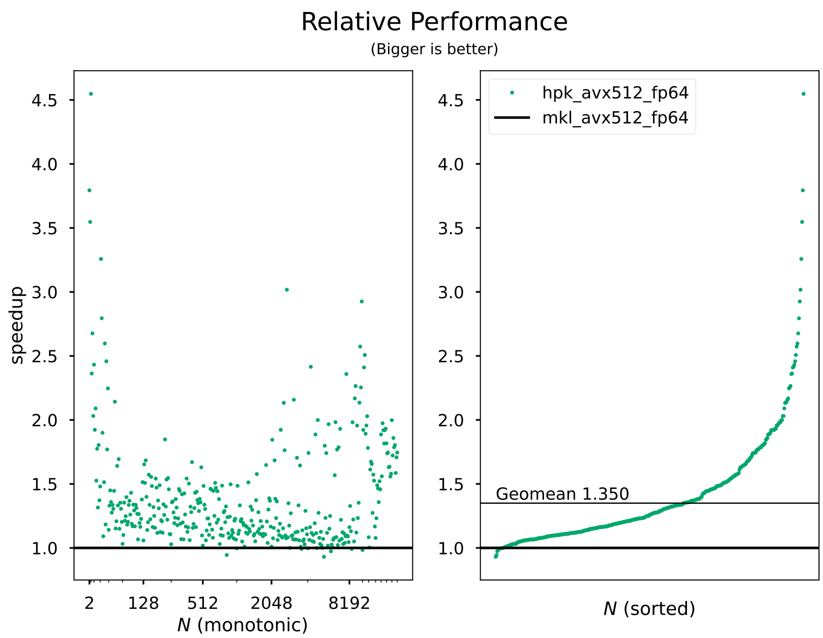



Fig. 19. Performance for AVX512 double precision vs. MKL

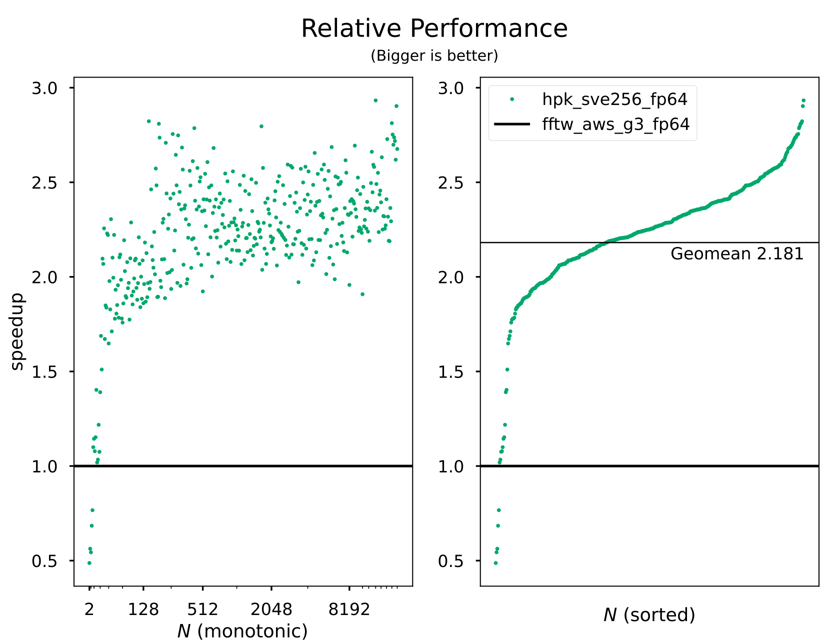



Fig. 20. Performance for AArch64 SVE256 double precision vs. FFTW

# 5 PYTHON

Results were obtained on `x86_64` using Debian 12, which provides Python 3.11.2, NumPy 1.24.2, and SciPy 1.10.1.

The SciPy FFT functions return a NumPy array as the result, so we measure only the performance of the Hpk out-of-place functions that do the same. We do two runs using the forward FFT and two using the backward, and we report the geometric mean of the four ratios.

Note that Hpk also provides `forwardCopy` and `backwardCopy`, which write results into an existing array to avoid allocating a return array. Furthermore, Hpk provides in-place FFT compute functions, which overwrite the input data with the results. These are significantly faster and are recommended when the input data is not subsequently needed.

## 5.1 Summary

Hpk is more accurate than SciPy:

| precision | ratio |
|---|---|
| float32 | 1.124 |
| float64 | 1.233 |

and has better performance:

| precision | ratio |
|---|---|
| float32 | 3.338 |
| float64 | 2.538 |

## 5.2 Accuracy

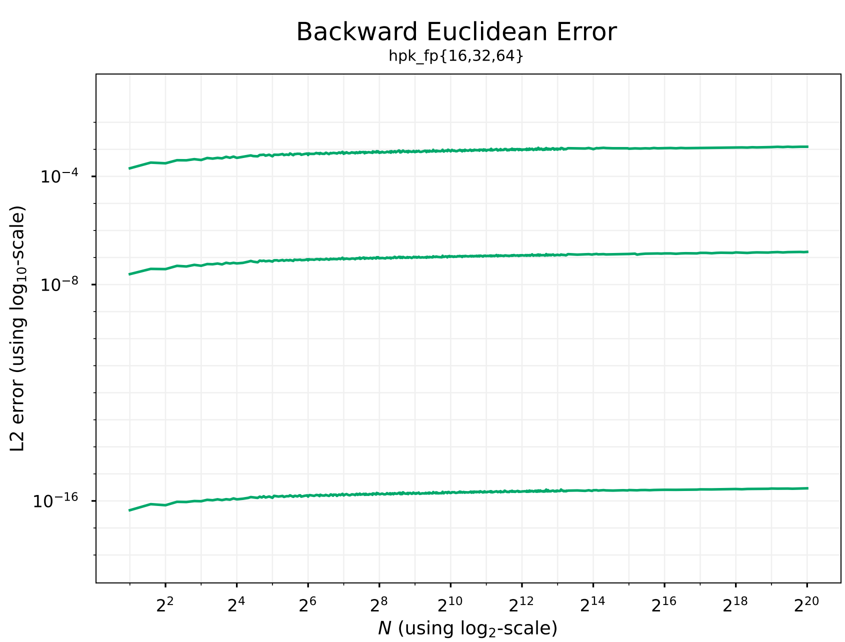



Fig. 21. Error for Hpk on a log-log scale for float16, float32, and float64.

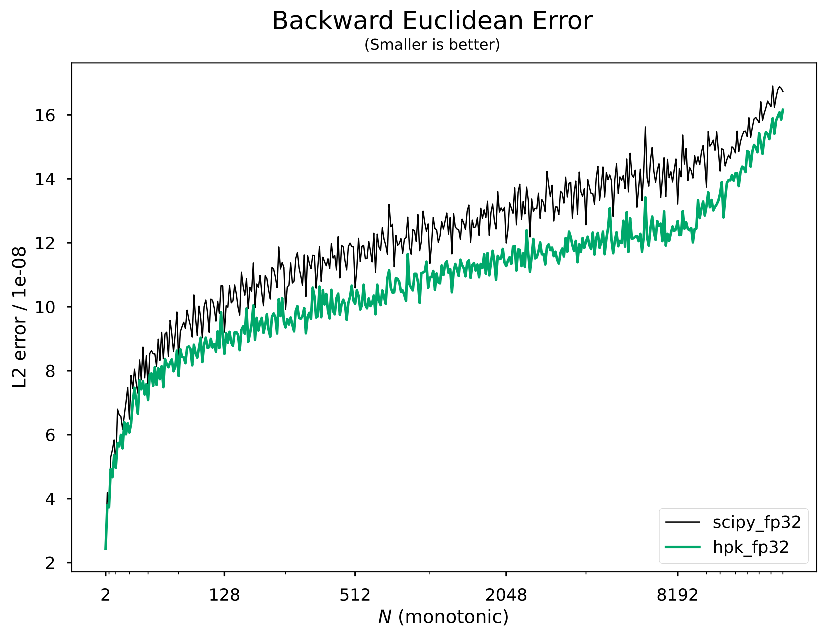



Fig. 22. Error for single precision vs. SciPy

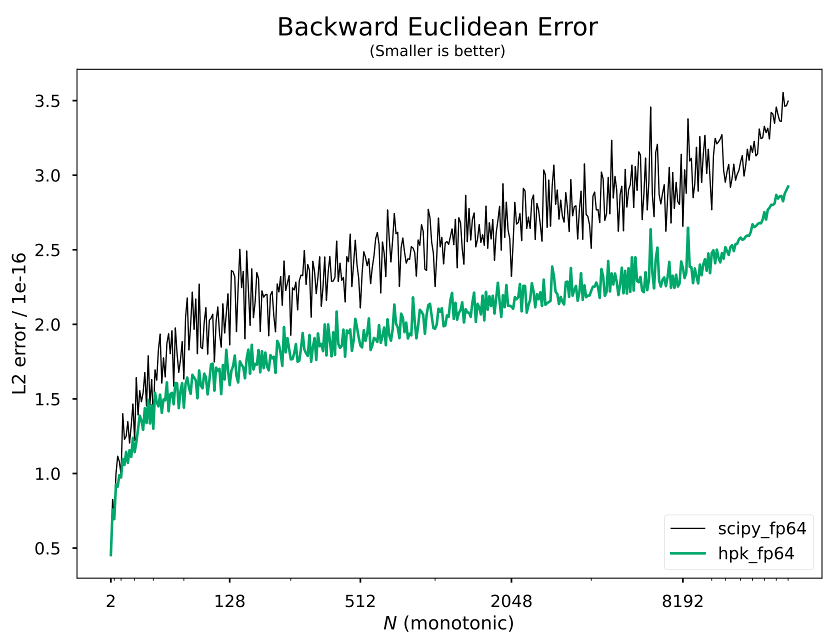



Fig. 23. Error for double precision vs. SciPy

## 5.3 Performance

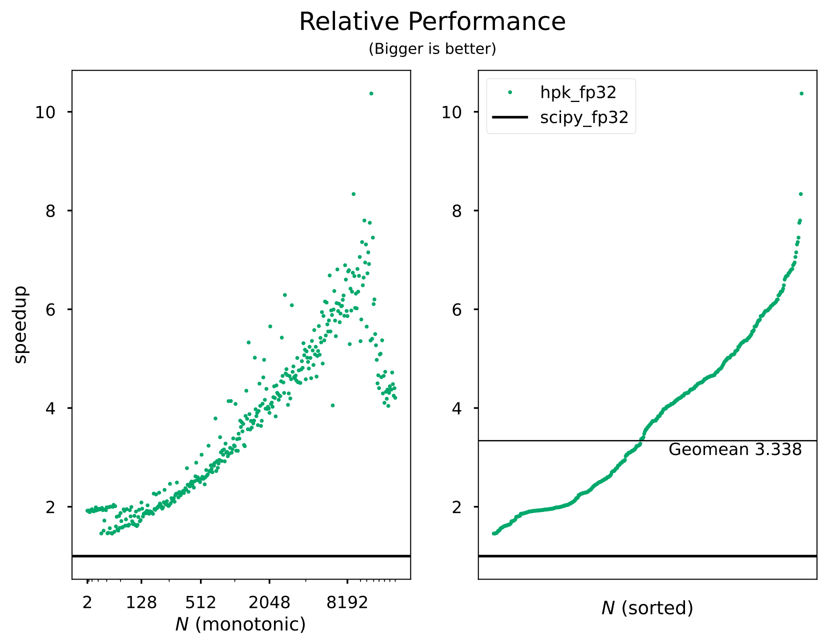



Fig. 24. Performance of single precision vs. SciPy

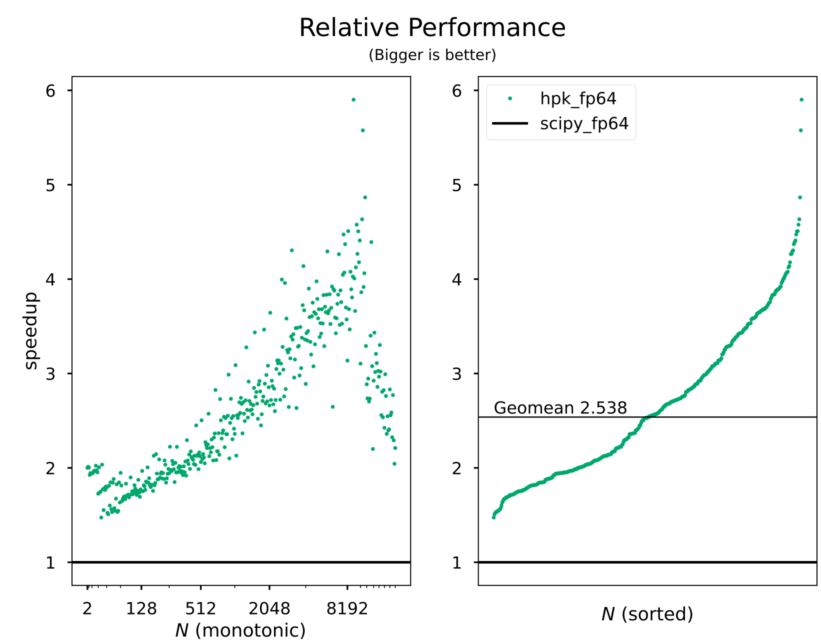



Fig. 25. Performance of double precision vs. SciPy

## 6 DOWNLOADING

High Performance Kernels for FFT, along with documentation, is available at `https://hpkfft.com`

## REFERENCES


[1] Paul Caprioli and Robby Jenkins. 2023. High Performance Kernels for FFT via Modern C++. https://doi.org/10.5281/zenodo.8253863

[2] Matteo Frigo, Steven G. Johnson, Paul Caprioli, et al. 2025. BenchFFT. https://bitbucket.org/hpkfft/benchfft/src/master/

[3] High Performance Kernels LLC et al. 2025. PyBenchFFT. https://bitbucket.org/hpkfft/pybenchfft/src/master/


Table 1. FFT lengths for half precision experimental results

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 24 | 60 | 108 | 180 | 256 | 360 | 512 | 648 | 900 | 1080 | 1440 | 1920 | 2400 | 3240 |
| 8 | 32 | 64 | 120 | 192 | 288 | 384 | 540 | 720 | 960 | 1152 | 1500 | 1944 | 2700 | 4096 |
| 12 | 36 | 72 | 128 | 216 | 300 | 432 | 576 | 768 | 972 | 1200 | 1536 | 2048 | 2916 | |
| 16 | 48 | 96 | 144 | 240 | 324 | 480 | 600 | 864 | 1024 | 1296 | 1620 | 2160 | 3000 | |

Table 2. FFT lengths for single and double precision results

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 32 | 78 | 144 | 234 | 360 | 528 | 800 | 1210 | 1764 | 2560 | 3584 | 5184 | 7000 | 12000 | 98304 |
| 3 | 33 | 80 | 150 | 240 | 375 | 540 | 840 | 1225 | 1792 | 2592 | 3600 | 5292 | 7056 | 14000 | 114688 |
| 4 | 35 | 81 | 154 | 243 | 384 | 550 | 864 | 1232 | 1800 | 2600 | 3780 | 5376 | 7168 | 15000 | 122880 |
| 5 | 36 | 84 | 156 | 245 | 385 | 560 | 875 | 1260 | 1815 | 2640 | 3840 | 5400 | 7200 | 16000 | 131072 |
| 6 | 39 | 88 | 160 | 250 | 390 | 576 | 880 | 1280 | 1872 | 2688 | 3888 | 5488 | 7500 | 16384 | 147456 |
| 7 | 40 | 90 | 162 | 252 | 392 | 588 | 896 | 1296 | 1920 | 2700 | 3920 | 5500 | 7680 | 17500 | 163840 |
| 8 | 42 | 91 | 165 | 256 | 396 | 600 | 900 | 1300 | 1944 | 2704 | 4000 | 5600 | 7776 | 18000 | 180224 |
| 9 | 44 | 96 | 168 | 260 | 400 | 624 | 924 | 1320 | 1960 | 2744 | 4032 | 5625 | 7840 | 20000 | 196608 |
| 10 | 45 | 98 | 175 | 264 | 405 | 630 | 936 | 1344 | 1980 | 2800 | 4096 | 5760 | 7920 | 21000 | 229376 |
| 11 | 48 | 99 | 176 | 270 | 420 | 640 | 945 | 1352 | 2000 | 2880 | 4116 | 5832 | 8000 | 24000 | 245760 |
| 12 | 49 | 100 | 180 | 280 | 432 | 648 | 960 | 1372 | 2016 | 2916 | 4200 | 5880 | 8064 | 29400 | 262144 |
| 13 | 50 | 104 | 182 | 288 | 440 | 660 | 968 | 1400 | 2048 | 2970 | 4320 | 6000 | 8100 | 32000 | 294912 |
| 14 | 52 | 105 | 189 | 294 | 441 | 672 | 972 | 1440 | 2080 | 3000 | 4400 | 6048 | 8192 | 32768 | 327680 |
| 15 | 54 | 108 | 192 | 297 | 448 | 675 | 990 | 1500 | 2100 | 3024 | 4480 | 6144 | 8400 | 36864 | 360448 |
| 16 | 55 | 110 | 195 | 300 | 450 | 700 | 1000 | 1512 | 2160 | 3072 | 4500 | 6272 | 8640 | 38400 | 393216 |
| 18 | 56 | 112 | 196 | 308 | 462 | 720 | 1008 | 1536 | 2200 | 3120 | 4536 | 6300 | 8748 | 40960 | 458752 |
| 20 | 60 | 117 | 198 | 312 | 468 | 728 | 1024 | 1540 | 2205 | 3136 | 4608 | 6400 | 9000 | 45056 | 491520 |
| 21 | 63 | 120 | 200 | 315 | 480 | 729 | 1040 | 1560 | 2240 | 3200 | 4704 | 6480 | 9072 | 49152 | 524288 |
| 22 | 64 | 125 | 208 | 320 | 490 | 735 | 1050 | 1568 | 2250 | 3240 | 4725 | 6561 | 9216 | 53248 | 589824 |
| 24 | 65 | 126 | 210 | 324 | 495 | 756 | 1080 | 1584 | 2268 | 3300 | 4800 | 6600 | 9240 | 57344 | 655360 |
| 25 | 66 | 128 | 216 | 330 | 500 | 768 | 1100 | 1600 | 2304 | 3360 | 4860 | 6720 | 9360 | 61440 | 720896 |
| 26 | 70 | 130 | 220 | 336 | 504 | 770 | 1120 | 1620 | 2352 | 3375 | 4900 | 6750 | 9600 | 65536 | 786432 |
| 27 | 72 | 132 | 224 | 343 | 512 | 780 | 1152 | 1680 | 2400 | 3456 | 5000 | 6804 | 9720 | 73728 | 917504 |
| 28 | 75 | 135 | 225 | 350 | 520 | 784 | 1176 | 1694 | 2500 | 3500 | 5040 | 6860 | 9800 | 81920 | 983040 |
| 30 | 77 | 140 | 231 | 352 | 525 | 792 | 1200 | 1728 | 2520 | 3528 | 5120 | 6912 | 10000 | 90112 | 1048576 |